2.3 Business Apps (*continued*)
How to do the problems:
STEP 0: Read question. Identify the
    function(s) in the question.

STEP 1: Find/simplify the functions.
    (Using 2.2 notation skills)

STEP 2: Roughly sketch the function.

STEP 3: Translate the question:
  (a) About shape/vertex?
  (b) About solving/quad formula?
  (c) Special business application?

STEP 4: Did you answer the
question?!

Entry Task:
2.3/9(c): Let price be p = -0.16q + 24q
and cost is TC(q) = 6q + 175. Give the
longest interval on which TR and
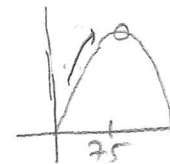Profit are both increasing.

$$TR(q) = -0.16q^2 + 24q \quad \leftarrow \text{PRICE} \cdot \text{QUANTITY}$$

STOPS INCREASING AT VERTEX

$$q = -\frac{b}{2a} = -\frac{24}{2(-0.16)} = 75$$
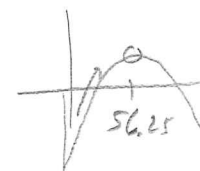
FROM   O   TO   75

$$P(q) = -0.16q^2 + 24q - 6q - 175 = -0.16q^2 + 18q - 175$$

STOPS INCREASING AT VERTEX

$$q = -\frac{18}{2(-0.16)} = 56.25$$

FROM   O   TO   56.25

BOTH   INCREASE FROM $\boxed{\text{O TO } 56.25}$

One more notation example: 1 ITEM ⟺ $q = 0.01$ HUNDRED ITEMS

Let

$$TR(x) = -2x^2 + 50x$$

$$TC(x) = 3x + 10$$

Assume q is in *hundreds* of items, and TR/TC are in *hundreds* of dollars.

Find the MR and MC formulas.

$$MR(x) = \frac{TR(x + 0.01) - TR(x)}{0.01} = \frac{[-2(x+0.01)^2 + 50(x+0.01)] - [-2x^2 + 50x]}{0.01}$$

$$= \frac{-2(x^2 + 0.02x + 0.0001) + 50x + 0.5 + 2x^2 - 50x}{0.01}$$

$$= \frac{-2x^2 - 0.04x - 0.0002 + 0.5 + 2x^2}{0.01} = \frac{-0.04x + 0.4998}{0.01}$$

$$\boxed{MR(x) = -4x + 49.98}$$ $/ITEM

$$MC(x) = \frac{TC(x + 0.01) - TC(x)}{0.01} = \frac{[3(x+0.01) + 10] - (3x + 10)}{0.01} = \frac{3x + 0.03 + 10 - 3x - 10}{0.01}$$

$$\boxed{MC(x) = 3}$$ $/ITEM

A Big Example: You are given

$VC(q) = 0.01q^3 - 0.135q^2 + 0.6075q$

$MC(q) = 0.03q^2 - 0.27q + 0.6075$

FC = 90 hundred dollars

q is in **hundreds of Objects**

VC is in **hundreds of dollars**

MC is in **dollars/Object** (as always)

Set up (do not compute) how to answer:

1. Find the cost to make the 326th item.

2. If the selling price is $30 per object, at what quantity is profit is zero?

3. For what quantities is TR = VC?

4. What is break even price (BEP)?

5. What is shutdown price (SDP)?

---

1 "mc at 325" = "CHANGE IN COST From 325 TO 326"

325 OBJECTS ⟹ q = 3.25 HUNDRED OBJECTS

$mc(3.25) = 0.03(3.25)^2 - 0.27(3.25) + 0.6075$

≈ 0.046875 $/ITEM   ABOUT 5 cents

---

2 $TR(q) = 30q \overset{?}{=} 90 + 0.01q^3 - 0.135q^2 + 0.6075q = TC(q)$

$\Rightarrow 0.01q^3 - 0.135q^2 - 29.3925q + 90 = 0$

(CUBIC) ↑   GIVE UP! ☹, WE DON'T LEARN TO SOLVE IN MATH 111.

$q \approx 3.0293$ & $q \approx 59.99$

---

3 $30q \overset{?}{=} 0.01q^3 - 0.135q^2 + 0.6075q$

÷q (ALSO CUBIC, BUT WE CAN DIVIDE EVERYTHING BY q!

$30 = 0.01q^2 - 0.135q + 0.6075$

$\Rightarrow 0 = 0.01q^2 - 0.135q - 29.3925$ { USE QUAD. FORMULA!

$q \approx -47.88$ & $q \approx 61.38$

---

4 5   SEE NEXT PAGE

Now let's forget the given market price
and do a general cost analysis.
What is break even price (BEP)?

$\underline{\text{LOWEST } y\text{-VALUE OF } AC}$ OR y-VALUE WHEN $AC = MC$

$AC(q) = \frac{90}{q} + 0.01q^2 - 0.135q + 0.6075$ OR

$\begin{pmatrix} \text{WE DON'T KNOW HOW TO} \\ \text{FIND THE LOWEST VALUE} \\ \text{FOR THIS IN MATH!!!} \end{pmatrix}$

$\frac{90}{q} + 0.01q^2 - 0.135q + 0.6075 \overset{?}{=} 0.03q^2 - 0.27q + 0.6075$

$\Rightarrow \frac{90}{q} = 0.02q^2 - 0.135q \Rightarrow 90 = 0.02q^3 - 0.135q^2$

$\Rightarrow 0 = 0.02q^3 - 0.135q^2 - 90$

SOLVER

$\Rightarrow x \approx 19.094$

WE DON'T KNOW HOW
TO SOLVE IN MATH!!!

y-VALUE $= AC(19.094) \approx 6.38914$ $\boxed{BEP = \$6.39 / ITEM}$

What is shutdown price (SDP)?

$\underline{\text{LOWEST } y\text{-VALUE OF } AVC}$ OR y-VALUE WHEN $AVC = MC$

$AVC(q) = 0.01q^2 - 0.135q + 0.6075$

QUADRATIC!

$q = -\frac{b}{2a} = -\frac{-0.135}{2(0.01)} = 6.75$

y-VALUE $= AVC(6.75)$

$= 0.01(6.75)^2 - 0.135(6.75) + 0.6075$

≈ 0.151875

$\boxed{SDP = \$0.15 / ITEM}$

OR

$0.01q^2 - 0.135q + 0.6075 \overset{?}{=} 0.03q^2 - 0.27q + 0.6075$

$0 = 0.02q^2 - 0.135q$ ← FACTOR

$0 = q(0.02q - 0.135)$ OR   QUADRATIC FORMULA

$q = 0$ OR $0.02q - 0.135 = 0$

$q = \frac{0.135}{0.02} = 6.75$

$AC(6.75) \approx 0.151875$

$\boxed{SDP = \$0.15 / ITEM}$

Example: *(Like 2.3(pt2)/ 1)*

Assume MR(x) = 30 − 2x and
$\qquad$ MC(x) = 4 + 5x
where x is in hundreds of items
and MR & MC are in dollars/item.

(a) What is the additional revenue if quantity changes from 300 to 301?

$$x = 3$$

$$MR(3) = 30 - 2(3) = 24 \quad \$/_{ITEM}$$

$$\boxed{\$24}$$ ADDITIONAL REVENUE
IF YOU SELL THE
301st ITEM.

(b) What quantity maximizes profit?

$$MR(x) \stackrel{?}{=} MC(x)$$

$$30 - 2x = 4 + 5x$$

$$26 = 7x$$

$$x = \frac{26}{7} \approx 3.7142857$$

$$\boxed{x = 3.71 \quad \text{HUNDRED ITEMS}}$$

(c) If $AC(q) = 4 + 2.5\,x + \dfrac{3}{x}$, what is the Break Even Price (BEP)?

y-VALUE OF AC & MC

WHEN $\quad AC = MC$

$$4 + 2.5x + \frac{3}{x} \stackrel{?}{=} 4 + 5x$$

$$\frac{3}{x} = 2.5x$$

$$3 = 2.5x^2$$

$$1.2 = x^2$$

$$x = \sqrt{1.2} \approx 1.095445\ldots$$

$$BEP = MC(1.095445\ldots)$$

$$\approx 9.477226\ldots$$

$$\boxed{\$9.48 / \text{ITEM}}$$

Example: *(Like 2.3(pt2)/3)*

If x is in thousand items, then

AVC(x) = 2x + 100   dollars/item

price = p = 1500 − 3x   dollars/item

FC = 2 THOUSAND DOLLARS

1. Find the VC(x), TC(x), and TR(x) formulas.

$$AVC(x) = \frac{VC(x)}{x} \iff x \cdot AVC(x) = VC(x)$$

So $VC(x) = 2x^2 + 100x$

$$TC(x) = 2x^2 + 100x + 2$$

$$TR(x) = 1500x - 3x^2$$

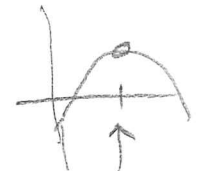2. What quantity maximizes profit? What price per item maximizes profit?

$$PROFIT = TR(x) - TC(x)$$

$$= (1500x - 3x^2) - (2x^2 + 100x + 2)$$

$$P(x) = -5x^2 + 1400x - 2$$

VERTEX!

$$x = \frac{-1400}{2(-5)} = 140$$

$\boxed{x = 140 \quad THOUSAND\ ITEMS}$

CORRESPONDING PRICE

$$p = 1500 - 3 \cdot 140 = \boxed{\$1080/ITEM}$$

Chapter 4 Motivation

We just spent 2 weeks discussing some of the algebra needed to study *linear* and *quadratic* **one variable problems**.

We will spend the next week discussing problems with **two variables** *(selling two products)*.

We will learn how to maximize and minimize two variable *linear* functions using the so-called **method of linear programming**.

Before we can do this, we learn to:
1. Find intersections of lines.(1.5)
2. Graph inequalities.   (4.1)

Example of 1.5 skills

Solve the system:

(i) $\quad 4x - y = 3$

(ii) $\quad 2x + 3y = 19$

STEP 1 SOLVE FOR ONE VARIABLE IN ONE EQUATION

$\text{(i)} \Rightarrow -y = 3 - 4x \Rightarrow y = -3 + 4x$

STEP 2 SUBSTITUTE IN FOR VARIABLE IN THE OTHER EQUATION.

$\text{(i)} \, \& \, \text{(ii)} \Rightarrow 2x + 3(-3 + 4x) \overset{?}{=} 19$

$\Rightarrow 2x - 9 + 12x = 19 \quad \leftarrow$ NOTE: ONLY ONE VARIABLE, SUCCESS!

$\Rightarrow \qquad 14x = 28$

$\boxed{x = 2}$

STEP 3 PLUG IN x "EVERYWHERE", SHOULD GET SAME y.

$\boxed{y = -3 + 4(2) = -3 + 8 = 5}$

STEP 4 CHECK! $x = 2, \; y = 5$

$\text{(i)} \quad 4(2) - (5) = 3 \checkmark$

$\text{(ii)} \quad 2(2) + 3(5) = 19 \checkmark$

Example from 1.5 with words (directly from homework):

Harry borrowed money from the bank and from his life insurance to start a business.
The bank loan has a 10% interest rate. The insurance has a 12% interest rate. If the total borrowed was $100,000 and the total interest in the first year is $10,700, how much did he borrow from each?

LABEL!!! Let x = "AMOUNT FROM BANK"

$\qquad\qquad\quad y$ = "AMOUNT FROM INSURANCE"

FACT 1 TOTAL BORROW = 100000

$\qquad \Rightarrow \quad x + y = 100000$

FACT 2 INTEREST = 10700

$\qquad \Rightarrow \quad 0.10x + 0.12y = 10700$

---

① $\Rightarrow y = 100000 - x$

① & ② $\Rightarrow 0.1x + 0.12(100000 - x) = 10700$ SUCCESS!

$\Rightarrow 0.1x + 12000 - 0.12x = 10700$

$\Rightarrow \qquad\qquad 12000 - 0.02x = 10700$

$\qquad\qquad\qquad -0.02x = -1300$

$\qquad\qquad\qquad x = \dfrac{-1300}{-0.02} = 65000$

$\qquad y = 100000 - 65000 = 35000$

$\boxed{\begin{array}{ll} x = \$65,000 & \text{from BANK} \\ y = \$35,000 & \text{from INSURANCE} \end{array}}$

CHECK!

① $65,000 + 35,000 = 100,000$ ✓

② $0.1 \cdot 65000 + 0.12 \cdot 35000$
$= 6500 + 4200 = 10,700$ ✓

Example of 4.1 Skills:
Graph the inequality:

$$4x - y \leq 5 \Rightarrow$$

OTHER WAYS TO WRITE THIS?

$$-y \leq 5 - 4x$$

$$\Rightarrow y \geq -5 + 4x$$

STEP 1 DRAW THE LINE. PLOT TWO PTS.

$$4x - y = 5$$

$$x = 0 \Rightarrow 4(0) - y = 5 \Rightarrow y = -5 \quad (0, -5)$$

$$y = 0 \Rightarrow 4x - (0) = 5 \Rightarrow x = \tfrac{5}{4} = 1.25 \quad (1.25, 0)$$

STEP 2 SHADE CORRECT SIDE

PICK A POINT ON ONE SIDE AND TEST IT!

OR $\quad y \geq -5 + 4x$

↑

"ABOVE" LINE $y = -5 + 4x$

FOR EXAMPLE:

$$(0, 0)$$

$$4x - y \leq 5 ?$$

$$\underbrace{4(0) - (0)}_{0} \leq 5 ? \boxed{YES!}$$

SHADE $(0,0)$ SIDE!

TEST? YES!

(1.25, 0)

(0, -5)